

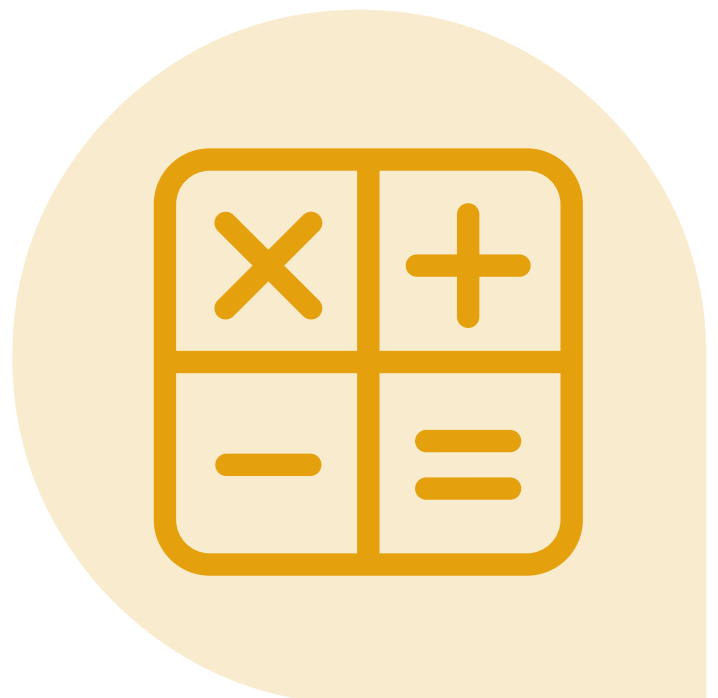


YEAR 11

MATHS

ADVANCED

LESSON THREE



THEORY

Quadratics

Last week we delved deeper into the topic of functions and started with investigating the linear function and its relevant applications. This week we will continue learning about functions, focusing specifically on a slightly more complicated function: the quadratic function. You may recall looking at quadratics in Lesson 1, and this is where we will start the lesson today.

In today's lesson we will:

- Recap key quadratic principles;
- Learn to graph parabolas from quadratic equations; and
- Learn to determine quadratic equations from given parabolas.

Quadratic Functions

Recall the general form of a quadratic equation is written as follows:

$$ax^2 + bx + c = 0$$

Where a, b, c are constants and $a \neq 0$

As such, recall that a quadratic function can be written as follows:

$$f(x) = ax^2 + bx + c$$

Considering $y = f(x)$, we may see the quadratic function written as follows:

$$y = ax^2 + bx + c$$

EXAMPLE QUESTION

Consider the equation $y = x^2 - 4x + 3$

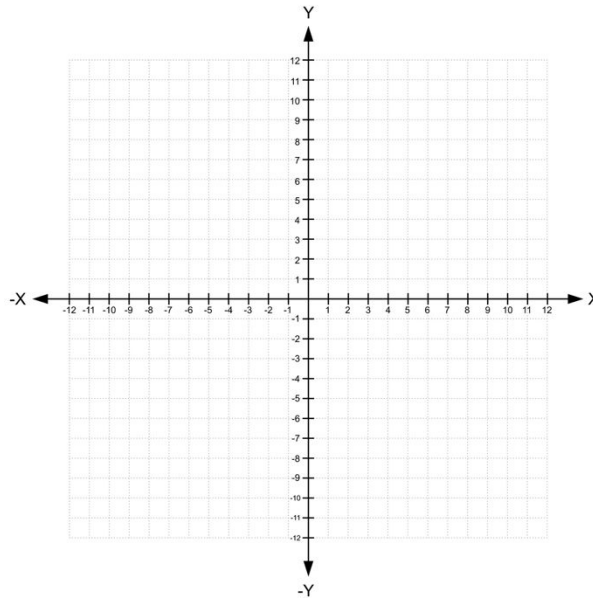
a. Fill in the below table of values.

(1 mark)

x	-1	0	1	2	3	4	5
y							

THEORY

- b. Plot the calculated values from Part a onto the following Cartesian plane. (1 mark)



What you have now drawn above is the graph of a quadratic function, otherwise known as a parabola. This is the focus of today's lesson.

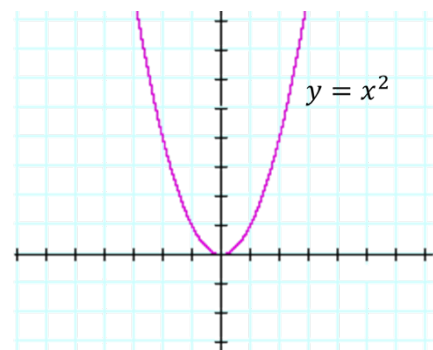
1.4.1 Identify the x-intercepts of a parabola whose quadratic function is expressed in factored form

Graphing a Parabola from a Quadratic Equation

The first thing we are going to learn is how to sketch a parabola. You just did it by finding points along the curve, plotting each point and drawing a curve through them, but that gets tedious pretty quickly. Luckily for us, there's a better way to sketch quadratic functions.

Every parabola has roughly the same shape, shown on the right, although their width and position are different. (You can plot a few different quadratic functions to check this. Use a table of values or a plotting software like Desmos or Geogebra).

Knowing this, we don't need to plot out every point. We can just sketch the shape of a parabola and label a few important features.



THEORY

The features we need to be able to find and label on our sketch are:

- x-intercepts
- y-intercepts
- Axis of symmetry
- Vertex

In the next few sections, we will go through each of these features in detail.

x-intercept(s)

We learnt in Lesson 1 that the x -intercepts of a function are the values of x where $y = 0$. These are also called zeroes of a function.

It follows that for the quadratic equation:

$$y = ax^2 + bx + c$$

the x -intercepts are solutions to:

$$ax^2 + bx + c = 0$$

So far in Mathematics Advanced you have learnt to solve quadratics in two ways:

1.
2.

We can use either method to help us find the x -intercepts of a quadratic function.

Quick Revision!

What is the quadratic formula?

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THEORY

EXAMPLE QUESTION

Consider:

$$x^2 - 5x + 6 = 0$$

- a. Solve the quadratic equation using the quadratic formula.

(1 mark)

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- b. Solve the quadratic equation using factorisation.

(1 mark)

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- c. What are the x-intercepts of the function $f(x) = x^2 - 5x + 6$?

(1 mark)

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THEORY

PRACTICE QUESTIONS

Using factorisation, find the x -intercepts for the following functions.

(2 marks each)

a. $f(x) = x^2 - 4x - 5$

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b. $f(x) = x^2 + 7x + 10$

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c. $f(x) = x^2 - 6x + 9$

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d. $f(x) = x^2 - 9$

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THEORY

e. $f(x) = 3x^2 + 15x - 18$

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f. $f(x) = 3x^2 - 2x$

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g. $f(x) = x^2 + 4$

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h. $f(x) = -x^2 + 2x + 1$

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THEORY

i. $f(x) = 2x^2 - 3x + 5$

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j. $f(x) = x^2 - 2x + 5$

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1.4.2 Use the discriminant to determine the number of x-intercepts on a parabola and justify its position in relation to the x-axis

The Discriminant

You may have noticed that some of the questions above are actually impossible to solve!

Which questions appeared to have no solution/x-intercepts?

In fact, we can actually deduce whether x -intercepts exist without solving the equation. We do this by using the discriminant.

Recall the discriminant to be:

$$\Delta = b^2 - 4ac$$

Where the pronumerals correspond to those in the general form of a quadratic equation:

$$ax^2 + bx + c = 0$$

THEORY

Using the discriminant, if:

- $x < 0$,
- $x = 0$,
- $x > 0$,

The reason for this comes back to the fact that the discriminant is found in the quadratic formula housed under the square root:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Going back to the following list we can justify the results further:

- $x < 0$, there are no x -intercepts
- $x = 0$, there is one x -intercept
- $x > 0$, there are two x -intercepts.....

Let's try use the discriminant for some of the previous practice questions.

EXAMPLE QUESTION

Noting the general form of a quadratic equation to be $ax^2 + bx + c = 0$, calculate the discriminant and determine how many x -intercepts each of the following functions has:

a. $f(x) = x^2 + 7x + 10$

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THEORY

b. $f(x) = x^2 - 6x + 9$

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c. $f(x) = x^2 - 2x + 5$

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PRACTICE QUESTIONS

For the following functions, identify the number of x -intercepts and if they exist, find the x -intercepts.

a. $f(x) = -x^2 + 2x - 3$

(1 mark)

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b. $f(x) = x^2 - 4x$

(3 marks)

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c. $f(x) = -3x^2 + 24x - 48$

(3 marks)

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THEORY

d. $f(x) = 2x^2 - 4x + 6$

(1 mark)

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e. $f(x) = x^2 - 4x + 3$

(3 marks)

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f. $f(x) = -2x^2 + 8x - 6$

(3 marks)

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NB: this booklet is only intended to show a rendering of a potential formatted booklet, theory pages have deliberately been omitted

HOMEWORK

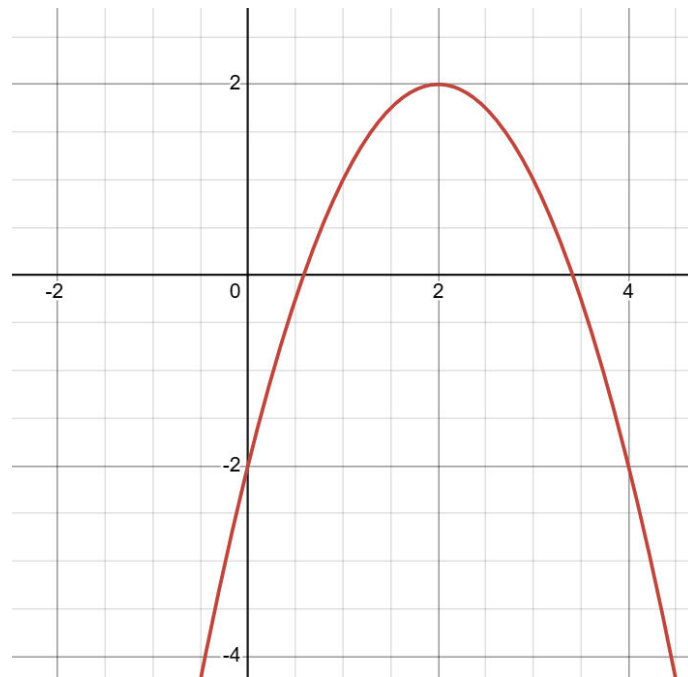
1. Which of the following correctly identifies the intercepts of the quadratic function $f(x) = x^2 - 6x + 5$?

- a. (1, 0), (5, 0)
- b. (3, 0), (2, 0)
- c. (1, 0), (0, 5), (5, 0)
- d. (3, 0), (2, 0), (0, 5)

2. Which of the following functions does not intersect with the x-axis?

- a. $f(x) = 4x^2 - 5x - 13$
- b. $f(x) = -x^2 + 4$
- c. $f(x) = x^2 - 10x + 25$
- d. $f(x) = x^2 - 3x + 4$

3. What is the equation of the following parabola?



- a. $f(x) = \frac{1}{3}((x - 2)^2 + 2)$
- b. $f(x) = -(x - 2)^2 + 2$
- c. $f(x) = -(x + 2)^2 + 2$
- d. $f(x) = (x - 2)^2 - 2$

HOMEWORK

4. Which of the following equations describe the same parabola?

i. $f(x) = (x - 6)(x + 2)$

ii. $f(x) = x^2 - 4x + 12$

iii. $f(x) = (x - 2)^2 + 8$

- a. i and ii
- b. i and iii
- c. ii and iii
- d. i, ii and iii

5. Given that a parabola passes through the points (0, 4), (-3, 1) and (1, 4), which of the following is the equation of its axis of symmetry?

- a. $x = 2$
- b. $x = -0.5$
- c. $x = 1$
- d. $x = 0.5$

6. A parabola has vertex (3, 8) and y-intercept (0, -1). Identify one other point that lies on the parabola.

(2 marks)

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HOMework

7. Convert the equation $f(x) = \frac{1}{5}(x - 4)(x + 6)$ to vertex-intercept form.

(2 marks)

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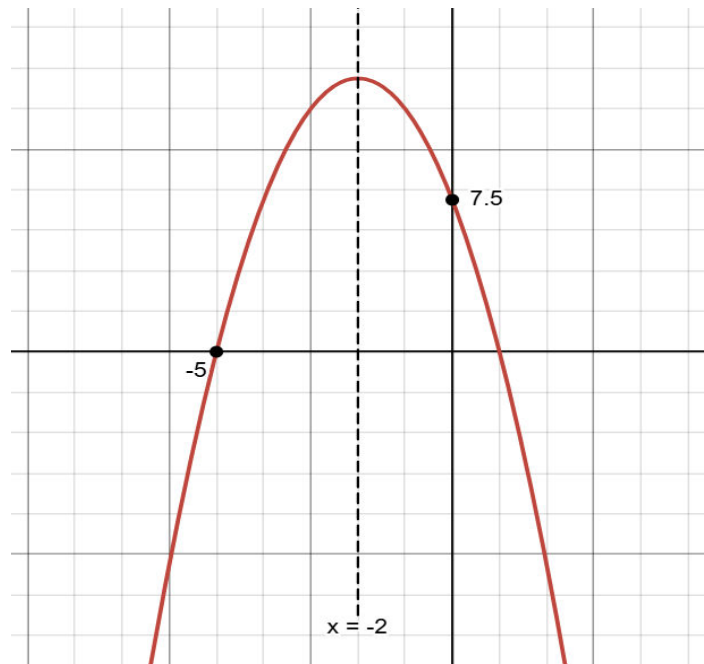
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HOMework

CHALLENGE QUESTION

Find the equation of the following parabola:

(4 marks)



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HW ANSWERS

Year 11 Advanced Lesson 2 Homework Answers

NB: this booklet is intended to show a rendering of a potential formatted booklet, HW answers have deliberately been omitted

